Additions to Cunningham's Factor Table of $n^4 + 1$

By A. Gloden

This note is the fulfillment of a plan to present in a readily accessible and concise form a complete list of additions to the factor tables of $n^4 + 1$ published by Cunningham [1], which give the prime factors (with certain omissions herein supplied) of all such integers not exceeding $1001^4 + 1$. Cunningham's factorizations were found with the aid of his tables [1] of solutions of the congruence

$$x^4 + 1 \equiv 0 \pmod{p}$$

for $p < 10^5$.

The subsequent tables of S. Hoppenot [2], A. Delfeld [3], and the writer [4] have provided an extension of these congruence tables to include all admissible primes between 10^5 and 10^6.

The factorizations presented in the present note have been extracted from a number of sources. The data corresponding to even values of $n \leqq 442$ and to odd values of $n \leqq 523$ have been published previously by M. Kraitchik [5] and N. G. W. H. Beeger [6]. The remaining data have appeared in a series of papers by the writer [7].

In Cunningham's table of factors of $n^4 + 1$ for $n = 2(2)1000$ there appear 97 incomplete entries. Of these, 66 are now identified as primes, corresponding to the following values of n:

320	442	526	616	742	800	952
328	466	540	624	748	810	962
334	472	550	628	758	856	966
340	476	554	656	760	874	986
352	488	556	690	768	894	992
364	492	566	702	772	912	996
374	494	568	710	778	914	
414	498	582	730	786	928	
430	504	584	732	788	930	
436	516	600	738	798	936	

Of the remaining 31 incomplete entries, 14 correspond to primes of the form

$$(n^4 + 1)/17,$$

namely, when $n = 648, 678, 682, 706, 746, 784, 790, 818, 842, 876, 882, 892, 954, 988$.

Furthermore, $(n^4 + 1)/41$ is a prime when $n = 888, 946$, and 998. Thus, there remain 14 omissions to be considered in Cunningham's table, for even values of n. These factorizations are now given *in extenso*.

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n	$n^4 + 1$
426	129553·254209
598	203569·628193
640	174289·962609
698	189017·1255801
714	216841·1198537
820	626929·721169
828	176041·2669977
844	246289·2060273
850	170873·3054937
880	290737·2062673
924	158993·4584689
938	809273·956569
980	780049·1182449
982	137593·6758489

In the companion table of factors of $n^4 + 1$, for $n = 1(2)1001$, there appear 82 incomplete entries, of which 68 have now been shown to correspond to primes of the form $(n^4 + 1)/2$. The related values of n are herewith listed:

403	471	539	623	719	821	895
405	477	543	639	721	829	913
415	479	551	643	725	833	917
419	487	561	649	745	843	919
431	503	567	657	761	845	931
445	505	573	677	769	855	963
449	513	579	681	795	857	965
453	517	605	701	805	879	997
455	523	607	703	811	883	
463	537	613	713	819	891	

Moreover, $(n^4 + 1)/2\cdot 17$ is prime for $n = 801, 859, 865, 869$, and 961; $(n^4 + 1)/2\cdot 41$ is prime for $n = 957$ and 981. In addition to these entries, it is now known that $(n^4 + 1)/2\cdot 17^2$ is prime when $n = 1001$.

Consequently, there remain only six entries to be considered, and for these the complete factorizations of $(n^4 + 1)/2$ are as follows:

n	$(n_4 + 1)/2$
565	157217·324089
595	137321·456353
685	147377·746969
889	505777·617473
893	17·104233·179441
941	132961·2948521

In conclusion, I should like to state that this paper was prepared as the result of a suggestion made to me by Dr. J. W. Wrench, Jr. that I consolidate my results and those of other researchers which complement the factorizations of $n^4 + 1$ published by Cunningham.

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5. M. KRAITCHIK, *Recherches sur la Théorie des Nombres*, v. 2, Gauthier-Villars, Paris, 1929, p. 116–117.
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7. A. GLODEN, "Compléments aux tables de factorisation de Cunningham," *Mathesis*, v. 55, 1945–46, p. 254–256; *ibid.*, v. 61, 1952, p. 49–50, 101, 305–306; v. 68, 1959, p. 172. See also *Intermédiaire des Recherches Mathématiques*, v. 4, 1948, p. 39.

On the Generation of All Possible Stepwise Combinations

By Gary Lotto

Conventionally, when all possible combinations of all possible subset sizes from a set of n are desired, a binary count is performed. Associating the units digit with the number 1, the two's digit with the number 2, the four's digit with the number 3, etc., the binary count 0001, 0010, 0011, 0100, 0101, 0110, 0111, 1000, etc., becomes associated with the combinations 1, 2, 12, 3, 13, 23, 123, 4, etc. This is useful in such procedures as the analysis of variance.

The above order of combinations requires that, when computing on data from one combination to the next, either (a) the calculation starts anew, or (b) if algorithms exist for generating a new function from the old one by single steps of either including or deleting a number from the combination, more than one step may be required. For example, we may go from the combination "2" to the combination "12" by "including 1." But going from "12" to "3" requires "deleting 1, deleting 2, and including 3."

Given, then, that a problem may be solved for some combination of k elements from the solution for the superset of $(k + 1)$ elements or the subset of $(k - 1)$ elements, is there an algorithm for generating all possible combinations which goes through the fewest recursions?

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